

# Nano Fluids and Heat Transfer Enhancement a Review

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**Abstract**— Superior performance and fast heat extraction coolants are crucial for increasing productivity and efficiency in many engineering fields. With ever reducing dimensions of equipment which are subjected to high heat loads the demand for high thermal conductivity coolants has reached all time high. Low thermal conductivity is major concern in current day cooling fluids. Maxwell in 19<sup>th</sup> century proposed the idea of increasing the thermal conductivity by adding solid particles to the coolant but because of the limited manufacturing capabilities, only particles with micrometer size could be produced. Suspensions with these types of particles caused abrasion of the tube wall and a substantial increase in the wall shear stress also the particle settling is a major drawback in such particle dispersed fluids. These problems can be overcome by using particles of less than 100 nm size and such particles are called nano particles and fluid in which these particles are dispersed are nano fluids. This review paper summarises the possible mechanisms of conduction heat transfer enhancement, thermal conductivity models, modelling the thermophysical properties for forced convection applications.

**Index Terms**— Aggregation, Brownian motion, Conduction, Forced Convection Heat Transfer, Nano Fluid, Nano Particle Clustering, Thermal Conductivity,

## 1 INTRODUCTION

Highly superior cooling fluids are needed for exceedingly fast heat dissipation in order to increase the efficiency of heat transfer equipment employed in industries such as microelectronics, transportation, manufacturing, metrology, and defense. Advancement in materials and manufacturing technologies lead to decrease in the sizes of engineering equipment with enhanced capabilities and generate high heat fluxes which is to be removed for effective working and longer life of the equipment. Heat fluxes of the order of 100 W/cm<sup>2</sup> are removed by aircooling but beyond it liquid cooling is to serve the task.

Conventional coolants have lower thermal conductivity an innovative approach to enhance the thermal conductivity and other heat transfer characteristics of a coolant is to add millimeter or micrometer sized particles to base fluid like water, ethyleneglycol etc. The major problem with suspensions containing millimeter- or micrometer-sized particles is the rapid settling of these particles. If the fluid is kept circulating to prevent particle settling, these particles would wear out pipes, pumps, and bearings. A solution to this is use of nano fluids. Nanofluids are uniformly dispersed suspensions of nanosized solid particles in a liquid. Current experimental research indicated that there is substantial enhancement of heat transfer with nano fluids [1].

Google scholar returns about 35500 results for the number

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of research articles published for the keyword nanofluids. The huge number of research articles indicates the enormous scale of interest among the researchers in these innovative fluids. Advantages of nanofluids over conventional solid-liquid suspensions as indicated by Choi, Eastman et al [2] and Saidur, Leong et al [3] are higher specific surface area, higher stability of the colloidal suspension, lower pumping power required to achieve the equivalent heat transfer, reduced particle clogging compared to conventional micro and millimeter sized particle suspensions, and better control of the heat transfer properties by changing the particle material, concentration, size, and shape.

The objective of this paper is to review the various mechanisms of thermal conduction enhancement, list out the models of thermal conductivity. Summarize the approaches in modelling the thermo physical properties in forced convection of nanofluids.

## 2 THERMAL CONDUCTIVITY ENHANCEMENT

A number of researchers as in table 1. Proposed different types of mechanisms for thermal conductivity enhancement, they are Brownian motion of nanoparticles, liquid layering of the base fluid surrounding nanoparticles, and nanoparticle aggregation.

### 2.1 Brownian Motion

Nano sized particles move randomly in the suspended fluid because of collision with the atoms or molecules of base fluid. Sometimes there is also collision among themselves this is called as Brownian motion. The effect of Brownian motion is that the heat transfers by diffusion in Brownian motion. The solid - solid heat transfer when two particles collide is the reason for thermal conductivity enhancement. The diffusion constant D is given by Stokes-Einstein formula:

$$D = \frac{K_B T}{3\pi\eta d} \quad (1)$$

$K_B$  = Boltzmann constant  
 $T$  = temperature  
 $\eta$  = viscosity of the fluid  
 $d$  = particle diameter

TABLE 1  
 MECHANISMS AND REFERENCES

Mechanism	References
Brownian motion	Wang, Xu, and Choi (1999)
	Kebllinski et al. (2002)
	Yu et al. (2003)
	Patel et al. (2003)
	Das, Putra, Thiesen et al. (2003)
	Koo and Kleinstreuer (2004)
	Bhattacharya et al. (2004)
	Jang and Choi (2004)
	Kumar et al. (2004)
	Patel et al. (2005)
	Prasher et al. (2005)
	Ren, Xie, and Cai (2005)
	Prasher, Bhattacharya, and Phelan (2006)
	Evans, Fish, and Kebllinski (2006)
	Beck, Sun, Teja et al. (2007)
	Shukla and Dhir (2008)
	Nie, Marlow, and Hassan (2008)
	Godson et al. (2010b)
	Kondaraju, Jin, and Lee (2010)
	Liquid layering
Yu and Choi (2003, 2004)	
Xue et al. (2004)	
Eastman et al. (2004)	
Xie, Fujii, and Zhang (2005)	
Shukla and Dhir (2005)	
Evans et al. (2008)	
Chandrasekar et al. (2009)	
Nanoparticle aggregation	Xuan, Li, and Hu (2003)
	Wang, Zhou, and Peng (2003)
	Prasher, Bhattacharya, and Phelan(2006)
	Zhu et al. (2006)
	Feng et al. (2007)
	Evans et al. (2008)
	Karthikeyan, Philip, and Raj (2008)
	Xu, Yu, and Yun (2006)
	Li, Zhang et al. (2008)
	Philip, Shima, and Raj (2008)

Main source [4]

## 2.2 Liquid Layering

Xie, Fujii, and Zhang etal [5] stated a liquid in contact with a solid interface is more ordered than the bulk liquid because Liquid molecules can form a layer around the solid particles and there by enhance the local ordering of the atomic structure at the interface region .The interaction and bonding between

the atoms of liquid and solid particle influences the size of the liquid layer which is usually of the order of several molecular distances. As the interaction gets stronger, evolution of a crystal like structure happens in the liquid surrounding the particle. These changes in the liquid structure have been shown to have significant effects on various properties including viscosity and thermal conductivity [6], [7]. The disorder in the liquid structure shrinks the effective collision mean free path to the order of one atomic distance. This is the logic behind why the solids exhibit better heat transfer properties than liquids. The increased order generated by liquid layering surrounding nanoparticles may have the ability to increase the liquid mean free path through which phonons can travel, causing an increase in thermal conductivity.

The possible enhancement of thermal conductivity by liquid layering is hindered by the interfacial resistance at the solid-liquid interface. This interfacial resistance  $R_K$ , also known as the Kapitza resistance [8], can arise from differences in phonon spectra in the two phases and from scattering at the interface between the phases as explained by Eastman etal [9].

## 2.3 Nano Particle Aggregation

Experimentally it has been proved that nanoparticles have the tendency to agglomerate in to clusters when suspended in the base fluid [10], [11]. Theoretically, nanoparticle clustering in to penetrating arrangements creates paths of lower thermal resistance that would have an extensive influence on the overall thermal conductivity [12], [13] and viscosity [14]. However, at lower particle volume fractions the thermal conductivity enhancement is not present because there are particleless zones in the liquid. The effective volume of a cluster is considered much larger than the volume of the particles due to the lower packing fraction of the cluster. Since, heat can be transferred rapidly within the clusters, the volume fraction of the highly conductive phase is larger than the volume of solid, thus increasing its thermal conductivity. The exact magnitude and effect of particle clustering in enhancing the thermal conductivity is still unknown as indicated by Özeriç and Kakaç et al [15]. The above three mechanisms ,which are infact concepts proposed by the various groups of researchers, no one was able to singleout the one which is responsible for thermal conductivity enhancement and the topic is still under debate [16], [17], [18].

## 3 THERMAL CONDUCTIVITY MODELS

The huge significance of thermal conductivity of nanofluids to thermal engineers worldwide led to large number of investigations and the output data is beyond that predicted by Maxwell theory [19]. Numerous mechanisms have been experimentally, theoretically, and numerically proposed to predict the scale and pattern of the enhanced thermal conductivity of nanofluids. However, the thermal conduction mechanisms of nanofluids are focused on the subject of much uncertainty and debate, due to the inconsistencies in thermal conductivity data reported by each group. The inconsistency of experimental data is because the various parameters affecting the thermal conductivity of nanofluids such as size, temperature, suspension quality, and pH, are not properly

controlled or characterized. High uncertainty of measurement devices is also one of the causes. Ji-Hwan Lee and Seung-Hyun Lee et al [20] shown that the conventional effective medium theories (EMTs) such as the Maxwell model are not sufficient to explain the novel thermal conduction behaviors of nanofluids. Basically, the EMT or effective medium approximation is an analytical approach to predict the effective properties of a mixture with the relative fractions and the properties of its components.

### 3.1 EMT Based models

The Maxwell Garnett model [21], as a representative of EMT, has been widely used as a comparison model for the effective  $k$  of nanofluids and as per the table .2

TABLE 2  
EMT BASED THERMAL CONDUCTIVITY MODEL

Authors	Formulation
Maxwell & Maxwell Garnett	$k_{eff} = \left\{ \frac{[k_p + 2k_{bf} + 2(k_p - 2k_{bf})\phi]}{[k_p + 2k_{bf} - (k_p - 2k_{bf})\phi]} \right\} k_{bf}$
Bruggeman	$\phi \left[ \frac{(k_p - k_{eff})}{(k_p + 2k_{eff})} \right] + (1 - \phi) \left[ \frac{(k_{bf} - k_{eff})}{(k_{bf} + 2k_{eff})} \right] = 0$
Hamilton & Crosser	$k_{eff} = \left\{ \frac{[k_p + (n-1)k_b - (n-1)(k_{bf} - k_p)\phi]}{[k_p + (n-1)k_{bf} + (k_{bf} - k_p)\phi]} \right\} k_{bf}$
Hashin & Shtrikman	$\left\{ \frac{[k_p + 2k_{bf} + 2(k_p - k_{bf})\phi]}{[k_p + 2k_{bf} - (k_p - k_{bf})\phi]} \right\} k_{bf} \leq k_{eff} \leq \left\{ \frac{[3k_f + 2\phi(k_p - k_{bf})]}{[3k_p - \phi(k_p - k_{bf})]} \right\} k_p$
Hasselman & Johnson	$k_{eff} = \left\{ \frac{[k_p(1+2\kappa) + 2k_{bf} + 2\phi(k_p(1-\kappa) - k_{bf})]}{[k_p(1+2\kappa) + 2k_{bf} - \phi(k_p(1-\kappa) - k_{bf})]} \right\} k_{bf}$
Nan et al.	$k_{eff,11} = k_{eff,22}$ $= \left\{ \frac{[2 + \phi[\beta_{11}(1-L_{11})(1+\cos^2\theta) + \beta_{33}(1-L_{33})(1-\cos^2\theta)]]}{[2 - \phi[\beta_{11}L_{11}(1+\cos^2\theta) + \beta_{33}L_{33}(1-\cos^2\theta)]]} \right\} k_{bf}$ $k_{eff,33} = \left\{ \frac{[1 + \phi[\beta_{11}(1-L_{11})(1-\cos^2\theta) + \beta_{33}(1-L_{33})(\cos^2\theta)]]}{[1 - \phi[\beta_{11}L_{11}(1-\cos^2\theta) + \beta_{33}L_{33}(\cos^2\theta)]]} \right\} k_{bf}$

Main source [22]

Where  $k$  and  $\phi$  denote the thermal conductivity and the volume fraction of nanoparticles, the subscripts bf, eff, and p indicate the base fluid, the nanofluid, and the nanoparticle, respectively. the model developed by Bruggeman et al [23] is a symmetrical EMT-based model to take into account the interactions between particles. Hamilton and Crosser [24] considered the arbitrary shape of nanoparticles in their empirical model based on EMT as in table. 2. The model is applicable when the thermal conductivity ratio of discontinuous phase ( $k_p$ ) to continuous phase ( $k_{bf}$ ) is greater than 100. in the formulation n is given as  $(3/\Psi)$ . Where  $n$  and  $\psi$  are the shape factor and the particle sphericity, which is expressed by the ratio of the sphere surface area (with the

same volume of the particle) to the given particle surface area. For sphere-shaped particles, the sphericity is 1 and the Hamilton–Crosser model approaches to the Maxwell model. Hashin and Shtrikman [25] developed the theoretical bounds of effective properties of a mixture based on the EMT. For the  $k_p > k_{bf}$ , the Hashin and Shtrikman (HS) lower bound is the same as the Maxwell model. With the HS bounds, Keblinski and Eapen et al. [26] demonstrated that almost all of thermal conductivity results of nanofluids fall within these bounds. Based on these results, they suggested that the thermal transport phenomena of nanofluids can be explained by the EMT theories without any new physics. However, the HS upper bound means that the base fluid is surrounded by the chain-clustered solid nanoparticles and this structure is not common in the well-dispersed dilute nanofluids. Hasselman and Johnson [27] developed a model of effective thermal conductivity with the consideration of the interfacial thermal resistance and the various shapes such as sphere, cylinder, and flat plate. The formulaton in table.2 is given for spherical shape where  $\kappa = a_k / a_p$  is a dimensionless  $\kappa$  parameter,  $a_k$  is the Kapitza radius defined as  $a_k = R_k k_{bf}$ ,  $a_p$  and  $R_k$  are the radius of nanoparticle and the Kapitza resistance. Nan et al [28] derived a generalized form of EMT taking into account numerous parameters such as the arbitrary shape and size of nanoparticles, the Kapitza resistance, and the orientation.

### 3.2 Nanolayer Based Models

The effective thermal conductivity model by taking in to account nanolayer concept was first proposed by Yu and Choi [29]. They reasoned that the nanolayer works as a bridge in between nano particle and fluid for thermal energy transfer the model is given in (2)

$$k_{eff} = \frac{k_{pe} + 2k_{bf} + 2(k_{pe} - k_{bf})(1 - \beta)^3 \phi}{k_{pe} + 2k_{bf} - (k_{pe} - k_{bf})(1 + \beta)^3 \phi} k_{bf} \quad (2)$$

Where

$$k_{pe} = \frac{[2(1 - \gamma) + (1 + \beta)^3(1 + 2\gamma)]\gamma}{-(1 - \gamma) + (1 + \beta)^3(1 + 2\gamma)} k_p \quad (3)$$

$$\gamma = \frac{k_{layer}}{k_p}, \beta = \frac{\delta}{a_p}$$

Where

- $k_{layer}$  = thermal conductivity of layer
- $\delta$  = thickess of nano layer
- $k_{pe}$  = equivalent thermal conductivity
- $a_p$  = radius of nano particle

Also, Xue and Xu [30] derived a model with a nanolayer concept by modifying the Bruggeman model. The results are as in (4)

$$\left[ 1 - \frac{\phi}{\chi} \right] \frac{k_{eff} - k_{bf}}{2k_{eff} + k_{bf}} + \left[ \frac{\phi}{\chi} \right] \frac{(k_{eff} - k_{layer})(2k_{layer} + k_p) - \chi(k_p - k_{layer})(2k_{layer} + k_{eff})}{(2k_{layer} + k_{eff})(2k_{layer} + k_p) + 2\chi(k_p - k_{layer})(k_{layer} - k_{eff})} = 0 \quad (4)$$

Where

$$\chi = \left[ \frac{a_p}{a_p + \delta} \right]^3$$

Xue et al also showed that the effective thermal conductivity estimated by yu and choi model is significantly not greater than that predicted by Maxwell theory. This indicates that the interfacial layer is not sufficient to describe the enhanced thermal conductivity of a nanofluids.

### 3.2 Nanoparticle Aggregation Based Models

Wang et al [31] derived an aggregation-based model depending on the EMT and fractal theory it is given by (5)

$$K_{eff} = \left\{ \frac{\left[ (1-\phi) + 3\phi \int_0^\infty [k_{cl}(a_{cl})n(a_{cl})] / [k_{cl}(a_{cl}) + 2k_{bf}] da_{cl} \right]}{\left[ (1-\phi) + 3\phi \int_0^\infty [k_{bf}(a_{cl})n(a_{cl})] / [k_{cl}(a_{cl}) + 2k_{bf}] da_{cl} \right]} \right\} k_{bf} \quad (5)$$

Where

$a_{cl}$  = equivalent radius of the cluster

$k_{cl}(a_{cl})$  = the effective thermal conductivity of nano particles obtained by the bruggeman model.

$n(a_{cl})$  = the radius distribution function.

### 3.3 Brownian Motion Induced Convection Models

Keblinski et al. [32] estimated the effect of Brownian motion on the thermal conductivity of a nanofluid by comparing the timescale of the diffusion of a nanoparticle with that of heat diffusion in the fluid. The characteristic timescale to cover a distance equal to the particle diameter  $d$  due to Brownian diffusion is

$$\tau_D = \frac{d^2}{6D} \quad (6)$$

Where  $D$  is the diffusion constant defined in (1). Similarly, the timescale for heat to move in the fluid by the same distance is

$$\tau_H = \frac{d^2}{6\chi} \quad (7)$$

Where  $\chi$  = thermal diffusivity, given by  $\chi = \frac{k}{\rho C_p}$

The ratio  $\frac{\tau_D}{\tau_H}$  is large and reduces to a small value when the

Nanoparticle size decreases from a few tens of a nanometer to atomic size, clearly indicating that heat transfer by conduction is much faster than Brownian motion based diffusion. Thus Keblinski et al concluded that the Brownian motion is too slow to transport an influential amount of heat through a nanofluid. Nanoconvection induced Brownian motion was first brought forward by Jang and Choi et al [33]. They presented that the

nano particle's Brownian motion agitates the neighbouring fluid molecules giving rise to nanoconvection. They developed a new model as in (8) for the prediction of effective thermal conductivity taking in to account the nano convection induced by Brownian motion

$$k_{eff} = k_{bf}(1-\phi) + \beta k_p \phi + C_1 \frac{d_{bf}}{d_p} k_{bf} \text{Re}_{d_p}^2 \text{Pr} \phi \quad (8)$$

$C_1$  = proportionality constant,  $\beta$  = Kapitza resistance/unit area

$\text{Re}_{d_p}$  = Reynolds number of nanoparticle

$$\text{Re}_{d_p} = \frac{C_{R.M.} d_p}{\nu}, C_{R.M.} = \frac{D_0}{l_{bf}} \quad (9)$$

$C_{R.M.}$  = random motion velocity of nano particle

$D_0$  = Diffusion Coefficient

$d_p$  = Particle Diameter,  $l_{bf}$  = liquid mean free path

above model is able to describe the temperature and size dependent behaviors of the effective  $k$  of nanofluids, unlike the EMT. Other models are summarized in table 3.

TABLE 3  
 NANO CONVECTION INDUCED BROWNIAN MOTION  
 THERMAL CONDUCTIVITY MODELS

Author	Formulation
Kumar et al.	$K_{eff} = \left[ 1 + C \left( \frac{2k_B T}{\pi \mu d_p^2} \right) \frac{\phi a_{bf}}{k_{bf} (1-\phi) a_p} \right] k_{bf}$ <p>where <math>C</math> is constant,  <math>k_B</math> = Boltzmann constant,  <math>\mu</math> = dynamic viscosity,  <math>a_{bf}</math> = radius of the base fluid</p> $k_{eff} = \left[ 1 + \frac{k_p}{k_{bf}} \left( \frac{\pi}{6 + (\alpha_{bf} \pi \eta d_p / 2 C k_B T)} \right) \left( \frac{6\phi}{\pi} \right)^{\frac{1}{3}} \right] k_{bf}$
Patel et al.	<p>where <math>C</math> = empirical constant  <math>T</math> = temperature  <math>\alpha_{bf}</math> = thermal diffusivity of base fluid</p> $k_{eff} = \left[ \frac{k_p + 2k_{bf} + 2(k_p - k_{bf})\phi}{k_p + 2k_{bf} - 2(k_p - k_{bf})\phi} \right] k_{bf}$ $+ \left[ (5 \times 10^4) \beta^1 \phi \rho_p c_{v,p} \sqrt{\frac{k_B T}{\rho_p d_p}} f(T, \phi \text{ etc}) \right]$
Koo and Kleinstreuer et al.	<p><math>c_{v,p}</math> and <math>f</math> are specific heat of the particle and a factorial function that depends on the temperature and the particle volume fraction.  <math>\rho_p</math> and <math>\beta^1</math> are density of the particle and the fraction of the liquid volume that moves with a nanoparticle.</p>

Main source [34]

Kumar et al. [35] derived a thermal conductivity model as in table. 3, consisting of two modes. For a stationary particle model, Fourier's equation is used. A moving particle is modeled by the kinetic theory of gases and the Stokes-Einstein formula. Patel et al. [36] developed a thermal conductivity model based on assumption of two parallel conduction paths for heat transfer. The first path is the heat conduction via the liquid medium and the second path is the heat propagation through particle conduction and random movement-induced microconvection both the above models can account for the effect of particle volume fraction, temperature, and particle size on the effective thermal conductivity of nanofluids.

### 3.1 Combined Mechanisms Based Models

Certain researchers like Xuan et al. [37] suggested model for the effective  $k$  of nanofluids taking in to account the Brownian motion and nanoparticle aggregation. The model is shown in (10). In the model there are two terms in the right hand side, the first one accounts for Maxwell theory and second term shows the enhancement in thermal conductivity due to Brownian motion of nano particle clustering.

$$k_{eff} = \left\{ \frac{k_p + 2k_{bf} + 2(k_p - k_{bf})\phi}{k_p + 2k_{bf} - (k_p - k_{bf})\phi} \right\} k_{bf} + \frac{\rho_p \phi c_{v,p}}{2\sqrt{k_B T / 3\pi a_d \mu}} \quad (10)$$

Prasher et al [38] proposed a model based on particle clustering and Brownian motion based microconvection their model is given in (11)

$$k_{eff} = (1 + A Re^m Pr^{0.33} \phi) \left\{ \frac{k_{ag} + 2k_{bf} + 2(k_{ag} - k_{bf})\phi_{ag}}{k_{ag} + 2k_{bf} - (k_{ag} - k_{bf})\phi_{ag}} \right\} k_{bf} \quad (11)$$

Where  $k_{ag}$  = thermal conductivity of aggregates

The assumptions made by the authors in developing the above model were that initially the nanoparticles were well dispersed in the base liquid and aggregation increases over time. the volume fraction  $\phi$  is split in two components as in (12)

Where,

$$\phi_p = \phi_{int} \phi_{ag} \quad (12)$$

$\phi_{int}$  = the particles in the aggregates

$\phi_p$  = volume fraction of primary particles

$\phi_{ag}$  = the aggregates in the entire fluid

Taking in to account the particle clustering and liquid layering feng et al [39] have developed a model as in (13). They assumed that the heat is transported through a unit cell, which consists of a nanoparticle, base liquid, and a nanolayer.

$$k_{eff} = \left\{ (1 - \phi_e) \frac{[k_{pe} + 2k_{bf} + 2(k_{pe} - k_{bf})(1 - \beta)^3 \phi]}{[k_p + 2k_{bf} - (k_{pe} - k_{bf})(1 - \beta)^3 \phi]} \right\} k_{bf} + \phi_e \left\{ \left[ \left(1 - \frac{3}{2}\right) \phi_e \right] + \frac{3\phi_e}{\eta} \left[ \frac{1}{\eta} \ln \frac{a_p + \delta}{(a_p + \delta)(1 - \eta)} - 1 \right] \right\} k_{bf} \quad (13)$$

Where  $\phi_e$  is the equivalent volume fraction given by

$$\phi_e = \phi(1 + \beta^3) \text{ and } \beta = \delta / a_p \quad (14)$$

$\delta$  = thickness of nano layer

$k_{pe}$  = equivalent thermal conductivity

$a_p$  = radius of nano particle

## 4 THERMO PHYSICAL PROPERTIES OF NANOFUIDS

The heat transfer results whether it be dissipation or cooling efficiency by utilizing nanofluids in forced convection problem is profoundly influenced by the way the nanofluid effective properties like density, specific heat, thermal conductivity and viscosity are modelled. Infact, the large variation among the results published by various research groups can be due to the dependence of the thermal transport phenomenon on such properties. Moreover, the large variety of theories and correlations that have been proposed and developed for modelling these thermophysical properties will undeniably yield results, predictions, and conclusions that can be different as presented by Mansour et al. [40].

### 4.1 Density

Nanofluid density is typically obtained by measuring the volume and weight of the mixture. The particle volume fraction  $\phi$  can be estimated knowing the densities of both constituents [41] (Pak and Cho 1998)

$$\rho_{nf} = (1 - \phi)\rho_{bf} + \phi\rho_p \quad (15)$$

The subscripts  $nf$ ,  $bf$ ,  $p$  denote nanofluid, basefluid and nanoparticle respectively. By rearranging the above equation, one can therefore determine the volume fraction  $\phi$  of the mixture as below

$$\phi = \frac{\rho_{nf} - \rho_{bf}}{\rho_p - \rho_{bf}} \quad (16)$$

### 4.2 Specific Heat

The simple mixing theory based equation has been suggested by Pak and Cho [41], Jang and Choi [42], Maiga et al [43] and Hwang et al [44],

$$C_{p,nf} = (1 - \phi)C_{p,bf} + \phi C_{p,p} \quad (17)$$

Other group of researchers have used an alternate approach based on heat capacity concept which can be deduced from

the first law of thermodynamics

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_{bf} + \phi(\rho C_p)_p \quad (18)$$

$$C_{p,nf} = \frac{\phi(\rho C_p)_p + (1 - \phi)(\rho C_p)_{bf}}{\phi\rho_p + (1 - \phi)\rho_{bf}} \quad (19)$$

### 4.3 Thermal Conductivity

Hamilton-Crosser correlation, has been used by various authors for numerical modeling of confined flows using nanofluids.

$$k_{eff} = \left\{ \frac{\left[ k_p + (n-1)k_b - (n-1)(k_{bf} - k_p)\phi \right]}{\left[ k_p + (n-1)k_{bf} + (k_{bf} - k_p)\phi \right]} \right\} k_{bf} \quad (20)$$

### 4.4 Dynamic Viscosity

Nanofluids exhibit either Newtonian or non Newtonian behavior is given by several researchers based on their experiments at varying volume fractions of nanoparticles for example. Park and Cho considered 13 nm sized Al<sub>2</sub>O<sub>3</sub> particles and when these particles were dispersed in water the viscosity was almost 200 times that of water and the mixture exhibited shear thinning behaviour when the particle volume fraction exceeded 3%. He also presented similar findings with 27 nm sized TiO<sub>2</sub> particles suspended in water, the viscosity was 3 times more than that of water and the fluid mixture exhibited non-newtonian behavior when particle volume fraction exceeded 10%. Similarly, Das et al. [45] found that in their mixtures containing Al<sub>2</sub>O<sub>3</sub> nanoparticles dispersed in water, the suspensions showed Newtonian behavior up to 4%. In the investigations carried out by Kulkarni et al. [46] (2007), results revealed that for nanofluids consisting of copper oxide nanoparticles dispersed in a 60:40 propylene glycol-water mixture, the fluids exhibited a Newtonian behavior for the considered range of  $0 \leq \phi \leq 6\%$ . An equation was proposed by Brinkman [47] based on Einstein's equation for viscous fluids containing diluted, small, rigid, and spherical particles in suspension.

$$\frac{\mu_{nf}}{\mu_{bf}} = \frac{1}{(1 - \phi)^{2.5}} \quad (21)$$

Also batchelor [48] proposed an equation as in (22)

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\phi + 6.5\phi^2 \quad (22)$$

## 5 CONCLUSIONS

As the conventional EMTs fail to sufficiently describe the novel aspects of thermal conduction in nanofluids, a number of new mechanisms and models proposed have been presented. Almost all the models proposed including the combined mechanisms based ones require certain information

and assumptions for obtaining the effective thermal conductivity of nano fluids. For example, the thickness and properties of nanolayer should be required for the nanolayer-based models. The existence of nanoconvection effect is also not experimentally evaluated at the nanoscale. Thus, the validity of the mechanisms proposed is ambiguous. Additional experiments are to be carried out with proper characterization and control. Also the physics of thermal transport in nanofluids is to be studied with scientific detail to accurately know the anomalous thermal conductivity enhancement in such fluids. Hence when modelling nanofluid forced convection problems

Correct selection of effective thermophysical properties of nanofluid is quite essential for obtaining accurate answers. The large variation in results predicted by various groups of researchers is due to the exemplary number of correlations and theoretical expressions for multiple types of nanofluids with varying volume fractions. Hence there is an utmost need for the reserachers to properly justify and reason their correlations based on the physics of the application.

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